

Dissipationless behavior of asymptotic non-Markovian dynamics within structured environments

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We investigate the long-lived quantum coherence in the non-Markovian process of spontaneous decay of a two-qubit system subject to a local environment. The asymptotic population of the qubit is linked to the spectral density of the reservoir through a general functional relation between them. The dissipationless behavior of asymptotic processes, including the steady coherent dynamics with self-sustained oscillations and ultimate persistence of quantum correlations, are elaborated with respect to spectral parameters of the specified reservoirs, e.g., the Ohmic class and the photonic crystal. We expect these results to contribute towards reservoir engineering with the aim of enhancing stationary quantum coherence in noisy environments.

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Understanding the non-Markovian dynamics of dissipative processes induced by memory effects of the environment is a fundamental subject of open quantum systems [1, 2]. In general, the non-Markovian process differs from the Markovian one by exhibiting the non-exponential decay or even dissipationless behavior; it becomes evident in cases of low-temperature environments and strong system-reservoir couplings and the resulted dynamics of the system depends dramatically on the spectrum structure of the environment. The relevant physics of the non-Markovian dissipative dynamics in the strong-coupling regime has been studied in the field for many years [3–5]. Due to the complexity of multiple energy scales of the overall system, e.g., that of the system, the environment, and of mutual coupling between the system and the environment, it is generally a challenging task to characterize the relevance of the system dynamics with the reservoir spectra.

Intensive interests upon the non-Markovian dynamics have been stimulated by the progress of quantum information science [6] since the task of decoherence control, aiming at performing quantum manipulation with high accuracy under noisy environments, requires that the memory effect of the environment needs to be taken into account. The back action of the non-Markovian environment will prolong quantum coherence and even lead to distinct longtime behavior for the system from Markovian processes. For instance, partial inhibition of radiative decay of atomic excited states can occur in structured environments of photonic crystals [7, 8] and high-Q cavities [9], and the associated potentiality of entanglement preservation under noisy environments has been unveiled in recent literatures [10]. Modifying the property of the reservoir to reach the non-Markovian regime was shown to be practicable in many physical systems, e.g., in photonic crystal materials [11] and in hybrid systems composed of quantum dots or trapped ions coupled with

optically confined ultracold atomic gases [12]. It is hence a fascinating task to describe the character of the long-time dynamics in connection with the spectrum structure of the non-Markovian environment, for the purpose of reservoir engineering to enhance stationary quantum coherence in noisy environments.

In this Letter we expose explicitly the link between the asymptotic dynamics of the system and the spectrum structure of the non-Markovian environment for the spontaneous emission model. We establish a general functional relationship between the asymptotic excited-state population and the spectral density of the reservoir. This enables us to carry on detailed analyses upon how to engineer the spectral parameters optimally to achieve stationary coherent dynamics of asymptotic processes for various reservoirs, e.g., the Ohmic class and the photonic crystal. Furthermore, we probe the persistence of quantum correlations—by notions of entanglement and quantum discord—of a two-qubit system undergoing local non-Markovian dissipation. The trapping phenomena of the two quantities and their dependence on the spectral densities of the reservoir are displayed.

The model we are considering is composed of three parts: qubits A , B , and a local environment E coupling with A . Initially the environment is in the vacuum state and the total system is described as $\rho_{tot}(0) = \rho_{AB}(0) \otimes |\{0_k\}\rangle\langle\{0_k\}|$. In the rotating-wave approximation, the interaction between A and E assumes the form

$$H_{AE} = \omega_0 \sigma_+ \sigma_- + \sum_k [\omega_k b_k^\dagger b_k + (g_k b_k \sigma_+ + g_k^* b_k^\dagger \sigma_-)], \quad (1)$$

where ω_0 is the transition frequency between upper and lower levels $|\pm\rangle$ of the binary system A and σ_\pm are the raising and lowering operators. The field modes of the reservoir are described by the creation (annihilation) operators b_k^\dagger (b_k) with frequencies ω_k and g_k account for the coupling constants of them with A . It is known that the time evolution of the reduced density operator of the qubit A is given by [1]

$$\rho_A(t) = \begin{bmatrix} \rho_{++}(0)|c(t)|^2 & \rho_{+-}(0)c(t) \\ \rho_{-+}(0)c^*(t) & 1 - \rho_{++}(0)|c(t)|^2 \end{bmatrix}, \quad (2)$$

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where the amplitude $c(t)$ satisfies the integro-differential equation

$$\dot{c}(t) + i\omega_0 c(t) + \int_0^t c(\tau) f(t - \tau) d\tau = 0, \quad (3)$$

and the kernel function of the integral is given by $f(t - \tau) = \int_0^\infty J(\omega) e^{-i\omega(t-\tau)} d\omega$ with $J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_k)$ representing the spectral density of the reservoir. For particular forms of $J(\omega)$, e.g., the spectra of photonic crystals, the above equation allows a solution with non-vanishing $c(t)$ in the longtime limit. Notwithstanding, resolution to the system of previous methods, either via numerical calculations [13] or analytical approaches (say, by the Laplace transformation [7, 8, 14] and by replacing the reservoir modes with pseudomodes [15, 16]), was mainly focused on the time evolution of the population. Straightforward revelation upon the connection between the asymptotic qubit population and the reservoir spectrum has only been obtained for very few cases with particular forms of the spectral densities [8, 16].

To this goal, let us consider the eigensolution of the Hamiltonian of Eq. (1) in the single-excitation sector: $H_{AE}|\Phi_{BS}\rangle = E|\Phi_{BS}\rangle$, where the energy E is determined by the secular equation

$$\omega_0 - \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega = E, \quad (4)$$

and the eigenstate takes the form $|\Phi_{BS}\rangle = b|+, \{0_k\}\rangle + \sum_k b_k |-, 1_k\rangle$ with the coefficients obtained as

$$b = \left[1 + \int_0^\infty \frac{J(\omega)}{(\omega - E)^2} d\omega \right]^{-1/2} \quad (5)$$

and $b_k = \frac{g_k b}{E - \omega_k}$. Note that the described issue, accounting for the coupling between the discrete levels and the continuous spectrum, identifies a standard Friedrichs model [17] and the specified eigenvalue problem could be retrospectively to the work by von Neumann and Wigner [18]. With the presumption that $J(\omega) > 0$ holds for $\omega > 0$, Eq. (4) allows at most a single real root that is non-positive [19]. For the mentioned environment of photonic crystals, the existence of the solution of atom-photon bound states was known well [7, 8] and the secular equation (4) might allow multiple real roots in view that the medium could possess several band gaps. Here we consider the case that Eq. (4) possesses only a unique real root. In this case, a direct connection between the asymptotic population and the reservoir spectrum can be established in view of the facts that: i) Eqs. (4) and (5) determine a conclusive relation of b with $J(\omega)$: $b = b[\omega_0, J(\omega)]$; ii) the asymptotic population on the excited state is given by: $|c(t)|^2 \xrightarrow{t \rightarrow \infty} |c_\infty|^2 = b^4$.

To make clear about the result (ii), we express the excited state as $|+, \{0_k\}\rangle = b|\Phi_{BS}\rangle + \bar{b}|\Phi_D(0)\rangle$, where $|\Phi_D(0)\rangle$ represents the projective state of $|+, \{0_k\}\rangle$ over the complementary subspace orthogonal to $|\Phi_{BS}\rangle$ and

$\bar{b} = \sqrt{1 - b^2}$ is the corresponding probability. The dynamics generated by the Hamiltonian (1) yields that $|\Phi_D(t)\rangle = e^{-iH_{AE}t}|\Phi_D(0)\rangle$ which will decay entirely as $t \rightarrow \infty$. So the amplitude of the excited state in the longtime limit is contributed solely by the ingredient of it in $|\Phi_{BS}\rangle$, which leads promptly to that $|c_\infty| = b^2$. Note that this fact has ever been revealed in the literature [8] to describe incomplete decay of an atom in photonic band gap mediums.

The above disclosed facts (i) and (ii) suggest an implicit functional relationship between the asymptotic population and the spectral density function, $|c_\infty|^2 = b^4[\omega_0, J(\omega)]$, which renders a peculiar perspective to look into the character of long-lived quantum coherence of the non-Markovian dynamics. It enables one to characterize the dissipationless behavior of ρ_A and ρ_{AB} in connection with the details of any specified spectrum structure of the environment. Later on we will elaborate the issue by focusing on two sorts of typical reservoir spectra, the Ohmic class $J(\omega) = J_o(\omega)$ and the photonic crystal $J(\omega) = J_p(\omega)$ [see Eqs. (6) and (8), respectively]. Before presenting these details, we highlight that the nonvanishing c_∞ suggests an attractive limit cycle [20], i.e., a close dynamical trajectory of ρ_A with self-sustained oscillations as time approaches infinity. Because a stable limit cycle is definitely a phenomenon of coherent dynamics—it could never happen in the statistic equilibrium of Markovian processes—its peculiar existence can be viewed as a representative of the non-Markovianity of dissipative dynamics. A visual description of coherent evolution of the limit cycle is shown in Fig. 1, where the trajectory of ρ_A in Bloch space is illustrated for the dissipative process with an Ohmic reservoir. The orbit is shown spiralling into a stable horizontal cycle around the z -axis. The radius of the limit cycle is determined by $a_\infty = |c_\infty|a(0)$ where $a(0) = 2|\rho_{+-}(0)|$ is the module of the horizon components of the initial state vector of $\rho_A(0)$.

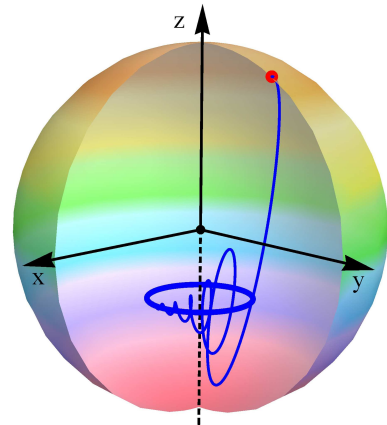


FIG. 1: Trajectory of time evolution of $\rho_A(t)$ in the Bloch space starting from an pure initial state $|\psi_A(0)\rangle = \cos(\pi/12)|+\rangle + i\sin(\pi/12)|-\rangle$. The spectral density of the reservoir is specified by $J(\omega) = J_o(\omega)$ [see Eq. (6)] with parameters $s = 5.5$, $\omega_c = 0.3\omega_0$ and $\eta_o = 0.08$.

From Eq. (4), it is evident that E will descend monotonically as the dissipation strength [e.g., $\eta_{o,p}$ in Eqs. (6) and (8) responsible for the reservoir of Ohmic class or photonic crystals, respectively] increases. The alteration of b in Eq. (5), relying on its contained integrand, is determined by the competition between the numerator and the denominator, both terms ascending with the dissipation strength. As a consequence, the asymptotic amplitude $|c_\infty|$ is generally not a simple monotone of the dissipation strength, but their relevance conditioned to details of $J(\omega)$.

Let us consider the reservoir described by the widely used Ohmic class spectra

$$J_o(\omega) = \eta_o \omega_c^{1-s} \omega^s e^{-\omega/\omega_c}, \quad (6)$$

where ω_c is the cutoff frequency and s is a parameter whose scope, $s < 1$, $s = 1$, $s > 1$, corresponds to sub-Ohmic reservoirs, Ohmic and super-Ohmic reservoirs, respectively. The secular equation (4) in this case allows an eigensolution of the bound state as long as the parameters satisfy $\eta_o^{-1} \leq \frac{\omega_c}{\omega_0} \Gamma(s)$ [21], where $\Gamma(s)$ is the gamma function. The functional relation of Eqs. (4) and (5) gives rise to

$$b(\eta_o, s, \omega_o/\omega_c) = \left[1 + \int_0^\infty \frac{\eta_o x^s e^{-x}}{(x - \kappa)^2} dx \right]^{-1/2}, \quad (7)$$

where $\kappa \equiv E/\omega_c$ is determined by the transcendental equation $\omega_o/\omega_c - \int_0^\infty \frac{\eta_o x^s e^{-x}}{x - \kappa} dx = \kappa$. Numerical calculation to the latter equation is needed to acquire κ for specified parameters $(\eta_o, s, \omega_o/\omega_c)$, with which the exact population $|c_\infty|^2 = b^4$ can be derived from Eq. (7).

We outline in Fig. 2 the characteristic of the population $|c_\infty|^2$ with respect to different zones of the spectral parameters. In the case of low ω_c/ω_o , a high value of $\eta_o \Gamma(s)$ indicates that the scope of the parameters (η_o, s) is relatively narrow to achieve a nonvanishing c_∞ . Note that there is a physical constraint of the coupling strength in order to validate the rotating-wave approximation for the model Hamiltonian (1) (reasonably $\eta_o \lesssim 0.1$ owing to $|g_k| \ll \omega_o$). The dependence of $|c_\infty|^2$ on η_o is shown in Fig. 2(a) with $\omega_c/\omega_o = 0.3$. Our calculation displays that high Ohmicity about $s \gtrsim 5.25$ is required, which may challenge the technology of the reservoir engineering. For the situation with high cutoff frequencies, the dependence of the population $|c_\infty|^2$ on the parameter s is shown in Fig. 2(b). Note that in the limit of $\omega_o/\omega_c \rightarrow 0$, the transcendental equation defines an implicit function $\kappa = \kappa(\eta_o, s)$, hence $b = b(\eta_o, s)$ according to Eq. (7). For $\eta_o = 0.08$, the maximum of the asymptotic population $|c_\infty|^2 \simeq 0.9$ is achieved at $s \simeq 2.34$.

For the second example we consider the structured environment of photonic band gap mediums with the spectral density given by [8, 14]

$$J_p(\omega) = \frac{\eta_p}{\pi} \frac{\omega_e^{3/2}}{\sqrt{\omega - \omega_e}} \Theta(\omega - \omega_e), \quad (8)$$

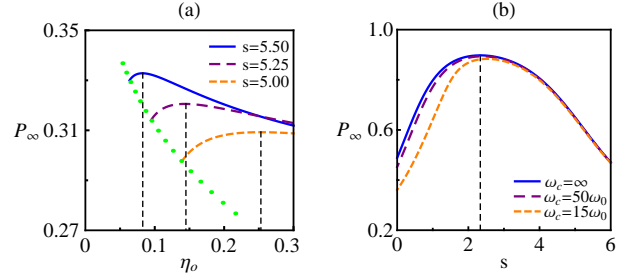


FIG. 2: Long-lived population $P_\infty = |c_\infty|^2$ in relation to the parameters of Ohmic class spectra. (a) P_∞ as a function of η_o with $\omega_c = 0.3\omega_o$, $s = 5, 5.25$, and 5.5 , respectively. The green-dotted line figures out the boundary determined by the critical condition $\eta_o \Gamma(s) = \omega_o/\omega_c$. For $s = 5.5$, the maximal $P_\infty = 0.33$ is achieved at $\eta_o = 0.08$. (b) P_∞ as a function of s with $\eta_o = 0.08$, $\omega_c/\omega_o = 15, 50$ and ∞ . In the limit $\omega_c/\omega_o \rightarrow \infty$, the maximal $P_\infty = 0.90$ is achieved at $s = 2.34$.

where ω_e is the band edge frequency, Θ is the Heaviside step function, and η_p accounts for the dissipation strength [22]. We have neglected the influence of the lower band by considering an atom with the transition frequency ω_o around ω_e . The specified integrals in Eqs. (4) and (5) for this case can be worked out analytically. In brief, Eq. (4) reduces to an algebraic equation $(\omega_o - E)\sqrt{\omega_e - E} = \eta_p \omega_e^{3/2}$ and the corresponding real root determines the bound-state energy $E = E(\eta_p)$. Consequently, according to Eq. (5) one obtains

$$b(\eta_p) = \left[1 + \frac{\eta_p \omega_e^{3/2}}{2(\omega_e - E)^{3/2}} \right]^{-\frac{1}{2}} = \left[1 + \frac{\omega_o - E}{2(\omega_e - E)} \right]^{-\frac{1}{2}}, \quad (9)$$

where the identity of the $E(\eta_p)$ -related algebraic equation has been used to achieve the latter equality. For fixed ω_o and ω_e , the peculiarity of the relevance $|c_\infty|^2 = b^4(\eta_p)$ is then clearly elucidated: i) for the case that the atomic level is inside the reservoir continuum ($\omega_o > \omega_e$), b is an increasing monotone of η_p ; ii) for the atomic level outside the continuum ($\omega_o < \omega_e$), b is a decreasing monotone of η_p ; iii) for the exact resonance case with $\omega_o = \omega_e$, the population becomes a constant $|c_\infty|^2 = 4/9$, which recovers the result of the previous literature [8].

Let us consider further the dynamics of the two-qubit density operator ρ_{AB} and the ultimate persistence of quantum correlations kept in the system. Straightforwardly, the time evolution of ρ_{AB} under the local dissipation can be described by the Kraus representation

$$\rho_{AB}(t) = \sum_{i=1}^2 \Gamma_i(t) \otimes I_B \rho_{AB}(0) \Gamma_i^\dagger(t) \otimes I_B, \quad (10)$$

where

$$\Gamma_1(t) = \begin{bmatrix} 0 & 0 \\ \sqrt{1 - |c(t)|^2} & 0 \end{bmatrix}, \quad \Gamma_2(t) = \begin{bmatrix} c(t) & 0 \\ 0 & 1 \end{bmatrix}. \quad (11)$$

The validity of the above result is justified by the fact that the expression of $\rho_A(t)$ in Eq. (2) can be recast as $\rho_A(t) = \sum_{i=1}^2 \Gamma_i(t) \rho_A(0) \Gamma_i^\dagger(t)$. One may notice that $c(t)$ will suffer a phase uncertainty for the longtime dynamics of the asymptotic process. It happens that the phase factor of $c(t)$ does not affect any measure of quantum correlations between A and B . This can be seen since if we replace $c(t)$ by its module $|c(t)|$ in $\Gamma_2(t)$, the yielded two-qubit state will differ from the original $\rho_{AB}(t)$ only by a local phase shift $u_A \otimes I_B$ with $u_A = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}$ and $\varphi = \arg[c(t)]$.

We employ two different notions of quantum correlations, entanglement and quantum discord, to illustrate the persistence of quantum coherence in the described dissipative process. Reported in Fig. 3 are the two quantities yielded from an initial Bell state subject to a local reservoir of the photonic crystal. The amount of entanglement of the two-qubit state is figured out via the measure of concurrence [23]: $C_{AB} = \max\{0, \sqrt{\lambda_1} - \sum_{i=2}^4 \sqrt{\lambda_i}\}$, where λ_i are eigenvalues of the matrix $\rho_{AB} \sigma_y \otimes \sigma_y \rho_{AB}^* \sigma_y \otimes \sigma_y$ in descending order. The discord of ρ_{AB} , according to definition of Ref. 24, is given by $Q_{AB} = \min_{\{A_k\}} \sum_k p_k S(\rho_B^k | \{A_k\}) + S(\rho_A) - S(\rho_{AB})$, where $S(\rho) \equiv -\text{tr}(\rho \log_2 \rho)$ is the von Neumann entropy. The first term in the definition, so called as conditional entropy, involves an optimization procedure over the projective measurement $\{A_k\}$ and a tractable approach is yet absent to deal with it for general two-qubit states. Favorably, we show in the below that for any pure input states, the discord as well as the concurrence of yielded states can be achieved analytically.

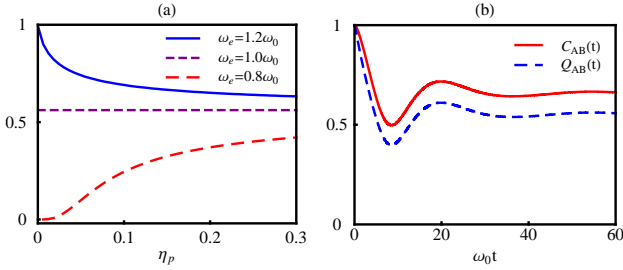


FIG. 3: Asymptotic behavior of quantum correlations of an initial state $|\psi_{AB}(0)\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)$ subject to a local structured reservoir of photonic crystals. (a) Quantum discord Q_{AB}^{∞} as a function of η_p with $\omega_e/\omega_0 = 0.8, 1.0$, and 1.2 . A constant $Q_{AB}^{\infty} = 0.56$ is obtained as $\omega_e = \omega_0$. (b) Time evolution of the discord as well as the concurrence for the case of $\omega_e = \omega_0$, where $\eta_o = 0.08$ is assumed.

Without loss of generality, we express the pure initial state as

$$|\psi_{AB}(0)\rangle = \alpha|+, \varphi_+\rangle + \beta|-, \varphi_-\rangle, \quad (12)$$

where α and β are positive real numbers satisfying $\alpha^2 + \beta^2 = 1$; $|\varphi_{\pm}\rangle$ are two normalized states of the qubit B but not necessarily to be orthogonal. According to Eq. (10), the state will evolve as

$$\rho_{AB}(t) = \alpha^2 \bar{c}^2(t) |-, \varphi_+\rangle \langle -, \varphi_+| + |\phi_{AB}\rangle \langle \phi_{AB}|, \quad (13)$$

where $\bar{c}(t) \equiv \sqrt{1 - |c(t)|^2}$ and $|\phi_{AB}\rangle = \alpha c(t) |+, \varphi_+\rangle + \beta |-, \varphi_-\rangle$. The concurrence is then obtained as $C_{AB}(t) = |c(t)| C_{AB}(0)$ with $C_{AB}(0) = 2\alpha\beta |\langle \varphi_- | \sigma_y | \varphi_+ \rangle|$ denoting the concurrence of $|\psi_{AB}(0)\rangle$. By noticing that the state $\rho_{AB}(t)$ is of rank two, we can work out its discord through a scheme proposed in Ref. 25. The result reads

$$Q_{AB}(t) = h(\lambda) + h(\lambda_A) - h(\lambda_{AB}) \equiv Q_{AB}(c(t)), \quad (14)$$

where $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary entropy function. All the contained parameters, λ , λ_A , λ_{AB} , hence the discord itself, are functions of the amplitude c . Their detailed expressions are shown as: $\lambda(c) = \frac{1}{2} [1 + \sqrt{1 - \bar{c}^2 C_{AB}^2(0)}]$, $\lambda_{AB}(c) = \lambda_A(\bar{c})$, and

$$\lambda_A(c) = \frac{1}{2} + \sqrt{\left(\frac{1}{2} - |\alpha c|^2\right)^2 + |\alpha\beta c \langle \varphi_+ | \varphi_- \rangle|^2}. \quad (15)$$

In the limit $t \rightarrow \infty$, a steady $|c_{\infty}|$ is reached, hence the discord $Q_{AB}(t) \rightarrow Q_{AB}^{\infty} = Q_{AB}^{\infty}[J(\omega)]$ (so does the concurrence C_{AB}^{∞}). For the specified spectra of photonic crystals, the relevance of Q_{AB}^{∞} with respect to η_o as well as the time evolution dynamics are manifested in Fig. 3. We point out that both the discord and concurrence of the output state (13) are monotonic functions of the upper-level population $|c|^2$, thus their longtime behavior is similar to that of $|c_{\infty}|^2$ described previously in connection with the parameters of various reservoir spectra.

In summary, the asymptotic non-Markovian dynamics of the spontaneous emission model was investigated. The dissipationless behavior of the model within various structured reservoirs has been elaborated via the functional relationship between the asymptotic excited-state population and the spectral function of the reservoir. Moreover, we have figured out the long-lived quantum correlations of a two-qubit system subject locally to the specified dissipative process. We expect these studies to contribute towards engineering of the reservoir spectra that aims at attaining steady quantum correlations under noisy environments.

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